**This work reflects my individual effort. I have not discussed the exam with anyone.**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

* Please answer all questions in the space provided and submit your solutions on Blackboard by 11:59 pm on October 10. Late submissions will be penalized by 20 Points.
* Please ask me in class on October 4 if you need any clarifications on the interpretation of the questions.
* Please present your work clearly and concisely so that I can follow your approach and arguments easily.
* You may use reference material, but your submission should strictly reflect your individual effort.
* **There should be no consultations or collaboration**.
* **Any collaboration carries a penalty of 25 Points.**

**Question 1. Constrained Optimization [12 points]**

A company produces and sells 3 grades of industrial solvents – A, B, and C. The selling price per gallon for the solvents are $6, $5, and $4 respectively. Because of demand limitations, the company can sell at most 120,000 gallons of solvent A, 240,000 gallons of solvent B, and 360,000 gallons of solvent C.

The solvents are produced by blending two types of liquid ingredients: *Ingredient1* and *Ingredient2*. The cost price per gallon for the ingredients are $4 for *Ingredient1* and $3 for *Ingredient2*. At most 360,000 gallons of *Ingredient1* and 320,000 gallons of *Ingredient2* are available.

Regulations allow a maximum percentage by volume of *Ingredient2* in each grade of solvent: 20% for A, 40% for B, and 60% for C.

For your convenience, the information presented above is summarized in the tables below:

|  |  |  |  |
| --- | --- | --- | --- |
| **Solvent grade** | **A** | **B** | **C** |
| **Selling price per gallon** | $ 6.00 | $ 5.00 | $ 4.00 |
| **Maximum quantity allowed (gallons)** | 120,000 | 240,000 | 360,000 |
| **Maximum % of *Ingredient 2* allowed** | 20% | 40% | 60% |

|  |  |  |
| --- | --- | --- |
|  | **Availability (gallons)** | **Price per gallon** |
| ***Ingredient1*** | 360,000 | $ 4.00 |
| ***Ingredient2*** | 320,000 | $ 3.00 |

The company must determine an optimal production plan so as to maximize their profits subject to the applicable constraints.

1. Formulate the problem as a linear program (**4 points**)

*Define the decision variables*:

*Specify the objective function:*

*Specify the constraints*:

1. Solve the linear program and report your optimal solutions (**4 points**)
2. What is the maximum profit attainable under an optimal plan? (**2 points**)

|  |  |
| --- | --- |
| Maximum Profit = | $ |

1. How many gallons of each ingredient should be used to produce each grade of solvent under this optimal plan? (**1 point**)

|  |  |  |  |
| --- | --- | --- | --- |
| **Quantity (in gallons)** | **A** | **B** | **C** |
| ***Ingredient1*** |  |  |  |
| ***Ingredient2*** |  |  |  |

1. How many gallons of each ingredient is used up under this optimal plan? (**1 point**)

|  |  |  |
| --- | --- | --- |
| **Quantity (in gallons)** | **Used** | **Available** |
| ***Ingredient1*** |  | 360,000 |
| ***Ingredient2*** |  | 320,000 |

1. At most how much should the company be willing to pay per gallon for *additional* quantities of the ingredients (over current availability)? Justify your answer. (**4 points**)

The maximum amount that the company should be willing to pay for each additional gallon:

|  |  |  |
| --- | --- | --- |
| *Ingredient1* : | $ | per gallon. |
| *Ingredient2* : | $ | per gallon. |

*Reasoning*:

# Question 2: Decision Analysis and Bayes Rule [13 points]

Two intrusion detection systems (IDS) – A and B – are available to block potentially harmful network events. Each IDS is prone to two types of errors. The table below summarizes the probability of these errors:

|  |  |  |
| --- | --- | --- |
| **IDS** | **False Alarm Error Probability** | **Miss Error Probability** |
| **A** | 0.05 | 0.02 |
| **B** | 0.01 | 0.06 |

* False Alarm error probability is defined as the conditional probability of blocking a harmless event.
* Miss error probability is defined as the conditional probability of not blocking a harmful event.

Historical data suggests that *4 percent* of the events are harmful.

1. Based on the information specified above, what is the conditional probability that:
2. An event blocked by IDS A is actually harmless? (**2 Points**)
3. An event not blocked by IDS B is actually harmful? (**2 Points**)
4. If the cost of not blocking a harmful event is 50 times the cost of blocking a harmless event, which IDS should a risk neutral rational decision maker use? Why? (**3 Points**)
5. We assumed that 4% of the events are harmful. At least how low must the percentage of harmful events be for a risk neutral rational decision maker to prefer IDS B? Assume that all other parameters remain as specified in (a) and (b). (**3 Points**)
6. We assumed that the ratio of the cost of not blocking a harmful event to the cost of blocking a harmless event is 50. At least how low must this ratio be for a risk neutral rational decision maker to prefer IDS B? Assume that all other parameters remain as specified in (a) and (b). (**3 Points**)